Q. Find the Hamiton's Equation of motion for Simple pendulum.

for simple pendulum

$$L = \frac{1}{2} m l^2 \dot{\theta}^2 + m g l \cos\theta$$

Now

$$H = \dot{\theta} P_\theta - L \quad \text{——} (1)$$

Now

$$P_\theta = \frac{\partial L}{\partial \dot{\theta}} = m l^2 \dot{\theta}$$

Equation (1) becomes.

$$H = \dot{\theta}(m l^2 \dot{\theta}) - \frac{1}{2} m l^2 \dot{\theta}^2 - m g l \cos\theta$$

$$H = m l^2 \dot{\theta}^2 - \frac{1}{2} m l^2 \dot{\theta}^2 - m g l \cos\theta$$

$$H = \frac{1}{2} m l^2 \dot{\theta}^2 - m g l \cos\theta$$

$$H = \frac{1}{2} \frac{m^2 l^4 \dot{\theta}^2}{m l^2} - m g l \cos\theta$$

$$\boxed{H = \frac{P_\theta^2}{2 m l^2} - m g l \cos\theta}$$

$$\dot{\theta} = \frac{\partial H}{\partial P_\theta}$$

$$\boxed{\dot{\theta} = \frac{P_\theta}{m l^2}} \Rightarrow \dot{P}_\theta = m l^2 \ddot{\theta}$$

$$\dot{P}_\theta = -\frac{\partial H}{\partial \theta}$$

$$\dot{P}_\theta = - m g l \sin\theta$$

$$\dot{p}_\theta = -mgl\sin\theta$$

$$\cancel{m}l^{\cancel{2}}\ddot{\theta} = -\cancel{m}g\cancel{l}\sin\theta$$

$$\ddot{\theta} = -\frac{g}{l}\sin\theta$$

$$\boxed{\ddot{\theta} + \frac{g}{l}\sin\theta = 0}$$

Hamilton's Principle :→ Hamilton's principle describe the motion of the mechanical system for which all the forces (except the force of constraints) are drivable from a generalized scalar potential, which may be function of co-ordinates velocity and time, Such system are known as monogenic system. This principle is also known as Integral principle.

Hamilton's principle

$$J = \int_{x_1}^{x_2} f(y, y', x)\, dx$$

for monogenic system Hamilton's principle is define as

$$I = \int_{t_1}^{t_2} L(q, \dot{q}, t)\, dt .$$

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Using Hamilton's principle, drive the Lagrangian Equation of motion.

Hamilton's principle $I = \int_{t_1}^{t_2} L(q, \dot{q}, t)\, dt.$

or

$$J = \int_{x_1}^{x_2} f(y, y', x)\, dx$$

where

$$f(y, y', x) = f(y_1(x), y_2(x) -- y_i(x), y_1'(x), y_2'(x) -- y_i'(x), x)$$

if the position change from $1 \longrightarrow 2$ then

$$J = \int_1^2 f(y_1(x), y_2(x) -- y_i(x), y_1'(x), y_2'(x) -- y_i'(x), x)\, dx$$

Suppose.

$$y_1(x, \alpha) = y_1(x, 0) + \alpha h_1(x)$$

$$y_2(x, \alpha) = y_2(x, 0) + \alpha h_2(x)$$

$$\vdots$$

$$y_i(x\alpha) = y_i(x, 0) + \alpha h_i(x)$$

Now

$$J(\alpha) = \int_1^2 f(y_1(x,\alpha), y_2(x, \alpha) -- y_i(x,\alpha), y_1'(x\alpha), y_2'(x,\alpha) -- y_i'(x,\alpha), x)\, dx$$

$$\partial J(\alpha) = \int_1^2 \frac{\partial f}{\partial y_1} \partial y_1 + \frac{\partial f}{\partial y_2} \partial y_2 \cdots \frac{\partial f}{\partial y_i} \partial y_i + \frac{\partial f}{\partial y_1'} \partial y_1' + \frac{\partial f}{\partial y_2'} \partial y_2'$$

$$\cdots + \frac{\partial f}{\partial y_i'} \partial y_i' + \frac{\partial f}{\partial x} \partial x \Big] dx.$$

$$\frac{\partial J(\alpha)}{\partial \alpha} d\alpha = \int_1^2 \left(\frac{\partial f}{\partial y_i} \frac{\partial y_i}{\partial \alpha} + \frac{\partial f}{\partial y_i'} \frac{\partial y_i'}{\partial \alpha} + \frac{\partial f}{\partial x} \frac{\partial x}{\partial \alpha} \right) d\alpha$$

$$\frac{\partial x}{\partial \alpha} = 0$$

b/c x is not contain α.

$$\frac{\partial J(\alpha)}{\partial \alpha} d\alpha = \int_1^2 \left(\frac{\partial f}{\partial y_i} \frac{\partial y_i}{\partial \alpha} + \frac{\partial f}{\partial y_i'} \frac{\partial y_i'}{\partial \alpha} \right) d\alpha\, dx$$

for maxima and minima $\frac{\partial J(\alpha)}{\partial \alpha} = 0$

$$\int_1^2 \left(\frac{\partial f}{\partial y_i} \frac{\partial y_i}{\partial \alpha} + \frac{\partial f}{\partial y_i'} \frac{\partial y_i'}{\partial \alpha} \right) d\alpha\, dx = 0$$

$$\int_1^2 \left(\frac{\partial f}{\partial y_i} \frac{\partial y_i}{\partial \alpha} \right) d\alpha\, dx + \int_1^2 \left(\frac{\partial f}{\partial y_i'} \frac{\partial y_i'}{\partial \alpha} \right) d\alpha\, dx = 0 \quad \text{--- (1)}$$

Now $$\int_1^2 \left(\frac{\partial f}{\partial y_i'} \frac{\partial y_i'}{\partial \alpha} \right) dx = \int_1^2 \left(\frac{\partial f}{\partial y_i'} \frac{d}{dx} \left(\frac{\partial y_i}{\partial \alpha} \right) \right) dx$$

$$= \frac{\partial f}{\partial y_i'} \frac{\partial y_i}{\partial \alpha} \Big|_1^2 - \int_1^2 \frac{d}{dx} \left(\frac{\partial f}{\partial y_i'} \right) * \frac{\partial y_i}{\partial \alpha} dx$$

$\because \frac{\partial f}{\partial y_i'} \frac{\partial y_i}{\partial \alpha} \Big|_1^2 = 0$ b/c all the point pass through the extreme point.

$$\int_1^2 \left(\frac{\partial f}{\partial y_i'} \frac{\partial y_i'}{\partial \alpha} \right) dx = - \int_1^2 \frac{d}{dx} \left(\frac{\partial f}{\partial y_i'} \right) \frac{\partial y_i}{\partial \alpha} \, dx$$

Now equation ① becomes.

$$\int_1^2 \left(\frac{\partial f}{\partial y_i} \frac{\partial y_i}{\partial \alpha} \right) d\alpha \, dx - \int_1^2 \frac{d}{dx} \left(\frac{\partial f}{\partial y_i'} \right) \frac{\partial y_i}{\partial \alpha} \, dx \, d\alpha = 0$$

$$\int_1^2 \left[\frac{\partial f}{\partial y_i} - \frac{d}{dx} \left(\frac{\partial f}{\partial y_i'} \right) \right] \frac{\partial y_i}{\partial \alpha} \, d\alpha \, dx = 0$$

but $\frac{\partial y_i}{\partial \alpha} d\alpha = dy$

$$\int_1^2 \left[\frac{\partial f}{\partial y_i} - \frac{d}{dx} \left(\frac{\partial f}{\partial y_i'} \right) \right] dy \, dx = 0$$

So

$$\left[\frac{\partial f}{\partial y_i} - \frac{d}{dx} \left(\frac{\partial f}{\partial y_i'} \right) \right] = 0$$

$$\boxed{\frac{d}{dx} \left(\frac{\partial f}{\partial y_i'} \right) - \frac{\partial f}{\partial y_i} = 0}$$

Let us do the following transformation:

$x \to t, \quad y_i \to q_i, \quad y_i' \to \dot{q}_i, \quad f \to L$

Then.

$$\boxed{\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = 0}$$

This is Lagrange's equation of motion.